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GREEDY ALGORITHMS FOR SPARSE AND REDUNDANT REPRESENTATIONS OF IMAGES

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Abstract

Obtaining an efficient representation of an image that provides accurate characterization of its features by linear representation methods is gaining significant attention. The sparse and redundant representations are one such powerful representations which code a large amount of the image information with only a few image samples. The sparse coding problem has two approaches namely solutions using greedy algorithms and solutions by solving convex relaxation of the problem. This paper presents an overview of significant greedy solution methods.

I. INTRODUCTION

There has been a huge progress in the area of image processing through the last years. This progress is mainly because of the type of image modeling used. Sparse and redundant interpretations of signals are being widely researched and have been effectively applied to a variety of applications like image de-noise, image compression , image compression such as image de-noising, image compression, image super resolution , object recognition etc.

The idea behind sparse and redundant representation modeling is to express a signal as a linear combination of few elements from a predefined dictionary. In sparse representation terminology the dictionary elements are referred to as atoms or columns[1]. In applications such as compression, the representation that is employed should be such that it should be capable of capturing a large part of signal information with only a few coefficients. Therefore the choice of dictionary becomes a matter of great concern.

Sparse coding is the way of determining the coefficients of representation based on both the given signal and the dictionary. This process is known as atom decomposition by pursuit algorithms.

An approximate solution is found through a search algorithm. Many pursuit techniques have been proposed over past decades with varying complexities and advantages.

The idea of obtaining a sparse representation of images dates back to 1993, when Mallat and Zhang proposed the concept [1]. The problem of spare approximation involves two procedures. They are sparse coding and sparse dictionary learning. The sparse coding is a task of identifying sparse vector. The two main approaches to estimate sparse vector are approximate solutions obtained using greedy algorithms and solving convex relaxation problem.

II. THE SPARSE MODEL

A linear combination of a base matrix $D \in R^{dxN}$ to represent a signal $y \in R^{dx1}$ is formulated as

$$y = Dx \qquad (1)$$

The above stated relation defines a linear system where $D = [d_1,d_2,d_3,....d_N] \in R^{dxN}$ is a dictionary of size dxN and $y \in R^{dx1}$ is a signal vector and it is assumed to be represented as linear combination of some columns of D. Every column of dictionary D is known as an atom. The dictionary D contains N linearly independent column vectors (atoms) which are basis vectors.

The multiplication of D by a sparse vector $x \in R^{Nx1}$ with k nonzero elements produces a linear combination of k atoms with varying weights generating the signal y as given in (1). The system $y = Dx$ is underdetermined system when $d < N$.

As the dictionary contains more than d non-zero atoms, the solution is not unique. Out of many solutions a solution needs to be identified that will be sparsest one. The sparsity of x is measured by l_0-norm $\|x\|_0$ which actually represents the number of nonzero components of x. The solution can be obtained by finding the solution to optimization problem,

$$\min\|x\|_0 \text{ subjected to } Dx = y \qquad (2)$$

The solution will be sparse when $\|x\|_0 \ll N$.

III. SPARSE SOLUTION BY GREEDY ALGORITHMS

Greedy algorithms are iterative methods of obtaining sparse approximation. In the greedy strategy approximation the aim is to solve the sparse representation method with l_0-norm minimization. But ,as the problem is an NP-hard problem , the greedy strategy provides only an approximate sparse representation solution. This section attempts to summarize the significant greedy techniques that have been proposed in literature.

A. Matching Pursuit Algorithm

The matching pursuit algorithm seems to be the oldest algorithms proposed by Mallat and Zhang[1] based on the concept projection pursuit strategies proposed by Friedman and Stuetzle[2].

The matching pursuit decomposes a signal into a linear expansion of waveforms which belong to a large and redundant dictionary of functions. From the redundant dictionary the waveforms are selected to match the signal structure.

Let $D = \{d_\gamma\} \epsilon\ R^{dxN}$ be the dictionary of vectors. The elements of dictionary D referred to as atoms are such that $\| d_\gamma \|=1$. Then the linear expansion of a vector sample f over a set of vectors selected from the dictionary is obtained by successive approximation of f with orthogonal projections on elements of dictionary D. Thus f can be decomposed as,

$$f = < f, d_{\gamma 0} > d_{\gamma 0} + Rf \quad (3)$$

where $d_{\gamma 0}\ \epsilon\ D$ and $< f, d_{\gamma 0} > d_{\gamma 0}$ represents orthogonal projection of f onto $d_{\gamma 0}$.The term Rf is the residual by using $d_{\gamma 0}$ to represent f .

As $d_{\gamma 0}$ and f are orthogonal, it leads to

$$\|f\|^2=\|<f,d_{\gamma 0}>\|^2+\|Rf\|^2 \quad (4)$$

The MP algorithm recursively finds out the atom that suits best to minimize the residual Rf. Almost the best match vector $d_{\gamma 0}$ is found when the following condition is satisfied.

$\|<f ,d_{\gamma 0} >\| \geq \alpha \sup | < f, d_\gamma >|$, where α is optimality factor; $0 < \alpha \leq 1$. (5)

The MP algorithm iteratively sub-decomposes the residual until the termination condition is satisfied. The n^{th} order residual is decomposed as

$$R^n f = < R^n f, d_{\gamma n}> d_{\gamma n} + R^{n+1} f \quad (6)$$

$R^n f$ and $d_{\gamma n}$ are orthogonal and hence

$$\|R^n f\|^2 = \|R^n f,d_{\gamma n}\|^2 + \|R^{n+1} f\|^2 \quad (7)$$

At k^{th} iteration, the vector sample is formulated as

$$f = \sum_{n=0}^{k-1} < R^n f, d_{\gamma n}> d_{\gamma n} + R^k f \quad (8)$$

When the residual is smaller than the pre-assumed value the f is approximated as

$$f \approx \sum_{n=0}^{k-1} < R^n f, d_{\gamma n}> d_{\gamma n} \quad (9)$$

Thus matching pursuit algorithm linearly decomposes the vector sample f into a sum of a few number of elements which are selected from a large and redundant dictionary to best match the residues.

Here the choice of vector $d_{\gamma 0}$ is not random but it is defined by a choice function that satisfies the condition $\|<f ,d_{\gamma 0} >\| \geq \alpha \sup | < f, d_\gamma >|$. The details are dealt by S. Mallat and Z. Zhang [1].

B. Orthogonal Matching Pursuit Algorithm

Orthogonal Matching Pursuit (OMP) is a refined version of the original Matching pursuit algorithm. The matching pursuit algorithm yields an approximation error which decreases with each iteration and hence the algorithm is guaranteed to converge. Being an iterative algorithm, it functions by projecting an initial signal and residuals on dictionary atoms during successive iterations. The specific base which is correlated to the residual is selected at each step. The selected best fitting basis is collected in a set and then coefficients are modified by projecting the residuals on this support collection. The algorithm ends on two conditions:

First, if the residual energy falls below a predefined threshold and second, if the support package exceeds a predefined sparsity limit.

Even though convergence is guaranteed after a finite number of iterations the results are suboptimal as proved by Y.C.Pati, Rezaiifar, R.; Krishnaprasad, P.S [3]. The OMP which is an improved version proves to converge at a faster rate than MP. The theoretical and empirical work on OMP is investigated in detail by J. A. Tropp and A. C. Gilbert [4].

For any known dimension dictionary of N elements, OMP approximates the projections onto the span of the dictionary elements in not greater than N steps. Also after a number of iterations, OMP provides the best approximation about the selected dictionary sub-set . This is accomplished by guaranteeing the full orthogonality of the backwards residue, a feature that is not available with MP algorithms. The proofs are given in [3].

C. Fast Matching Pursuit

The difficulty in real world implementation of MP and OMP is their computational complexity involved in sequential atom selection process in each iteration. To tackle this issue some fast processing approaches were proposed in [5]. The fast MP is a modified algorithm that uses atoms of anisotropic refinement to provide approximation capability. The experimental investigations show that the implementation is faster due to the usage of both sequential and parallel techniques.

The fast matching pursuit constructs a geometric dictionary in the first step and then employs two techniques, namely adaptive projection computation and M-fold atom selection, based on the dictionary cross-correlation, to decrease the complexity associated with the basic greedy algorithms. The dictionary is split into several sub-dictionaries, each one known as shape sub-dictionary. The projection computations are performed by estimating the cross-correlation between an atom d_γ and the atom selected during n^{th} iteration $d_{\gamma n}$. . Let $d_{si,n}$ be the best matching atom in the shape sub-dictionary S_i . At n^{th} iteration the incoherence is evaluated as

$$| < d_{si,n-1}, d_\gamma^{n-1} > | < \delta \quad (10)$$

where δ is a fixed threshold .If the above condition is true then projection updating of S_i is skipped. The above condition is true frequently since the atoms of the dictionary are spatially localized resulting in a reduced number of FFT operations.

The authors claim that image structures drawn by MP during consecutive iterations are spatial localized and mostly have no correlation. Therefore, with the aid of the estimated correlation the selection task is carried out in a simple way. The fast MP algorithm recursively finds an approximation by choosing atoms M rather than one atom at a time which in-turn significantly reduces the computational complexity.

D. Complementary Matching Pursuit

The algorithm resembles the original matching pursuit (MP) algorithms but performs the complementary action. In each iteration the MP selects only one atom for sparse approximation whereas the CMP algorithm deletes (N-1) atoms from the approximation in each iteration. It has been shown that due to complementary action the algorithm does not reduce the error in each iteration but results in faster convergence compared to MP [6].

E. Compressive Sampling Matching Pursuit

Compressive Sampling Matching Pursuit (CoSaMP)is one of the greedy pursuit algorithms proposed by D. Needell and J.A.Tropp [7]. Compressive sampling reconstructs signals and images from far less measurements. The CoSaMP is an improvement over OMP based on orthogonal matching pursuit (OMP).

The compressive sampling theory assumes that the applied signal is a linear signal sample function . The way samples are obtained is interpreted like a sampling matrix operating on the intended signal. Thus if p number of samples are collected or measured from a signal in C^N then the sampling matrix ψ has the dimension pxN. The condition that is used in acquiring minimum number of samples during compressive sampling is based on the fact that the matrix ψ should not map two different k-sparse signals to the same set of samples. Thus the collection of 2k columns from the sampling matrix must be a nonsingular or invertible. Violation of this condition may make the inverting sampling process unstable. Therefore a more robust solution to ensure minimum number of measurements proposed by Candes and Tao [8] emphasizes that geometry of sparse signals should be preserved under the action of sampling matrix ψ. Further, imposing restricted isometry on ψ ensures that the collection of r columns from ψ is nonsingular. Recovery by Compressive sampling matching pursuit theorem [7] is defined as follows.

Suppose that ψ is a matrix of measurement pxN that follows the restricted condition of isometry $\delta_{2k} < C$. Let u = $\psi\ x + e$ be an arbitrary signal sample vector contaminated with arbitrary noise e. The CoSaMP scheme produces a k-sparse approximation a for a given precision parameter, which satisfies

$$||x - a||_2 \leq c\ max\left\{\eta, \frac{1}{\sqrt{k}}||\mathrm{x} - x_{\frac{k}{2}} + ||e||_2\right\}$$

where $x_{\frac{k}{2}}$ is a k/2 sparse approximation to x.

Compressive sampling matching pursuit is proved to run faster compared to other greedy algorithms as it selects multiple components in each iteration The details are given by D. Needell and J.A. Tropp [7].

F. Regularized Orthogonal Matching Pursuit

The sparse signal recovery uses two different approaches namely iterative greedy methods and convex relaxation methods. While greedy methods being iterative in nature end up in sub-optimal solutions, the convex relaxation methods are numerically more complicated.

Regularized orthogonal matching pursuit algorithm has been developed as a modification of orthogonal matching pursuit (OMP)based on the study conducted by Holger Rauhut[14] on impossibility of uniform guarantees for orthogonal matching pursuit for natural measurement matrices.

The authors D.Needell and RomanVershynin [15]have pointed out the fact that the OMP has poor guarantees of exact recovery due to absence of a deterministic condition on measurement matrix. Motivated by these factors, the authors have proposed Regularized orthogonal matching pursuit and proved that exact recovery is possible given that the measurement matrix satisfies the restricted condition of isometry. The algorithmic details and experimental investigation on recovery of signals using ROMP are available in [15-16].

IV. CONCLUSION

The sparse and redundant representation of signals and images has emerged as a new method of signal acquisition and reconstruction. Being an indispensable part of emerging compressive sensing theory, the research in the field of sparse and redundant modeling and its various applications has high potential. This paper gives the glimpse of some important greedy pursuit algorithms.

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